11 WCEE
Paper No. 956. (quote when citing this article)
Eleventh World Conference on Earthquake Engineering
ISBN: 0 08 042822 3

NON-ITERATIVE COMPUTATION SCHEME FOR ANALYSIS OF NONLINEAR DYNAMIC SYSTEM BY FINITE ELEMENT METHOD

Hisakazu SAKAI[1)], Sumio SAWADA[2)]

1) Design Department, Wakachiku Construction Co., Ltd.,
23-18, Shimomeguro 2-Chome, Meguro-ku, Tokyo, 153, Japan.
2) School of Civil engineering, Kyoto University,
Yoshida-Honmachi, Sakyo-ku, Kyoto, 606-01, Japan.

ABSTRACT

A direct integration scheme to compute dynamic response of structure with nonlinear stiffness property is proposed. This method utilizes and combines the advantage of the Central Difference Method in accuracy and that of the Averaged Acceleration Method in unconditional stability, which does not require the iterative calculations. Numerical analyses are conducted to examine the usefulness and validity of the proposed method in nonlinear dynamic response analyses using Finite Element Method. The proposed method is stable in calculation due to the high damping property in higher frequency range and the high accuracy in lower frequency range.

KEYWORDS

direct integration approach; nonlinear analysis; finite element method; numerical stability; non-iterative scheme

INTRODUCTION

When we analyze the dynamic response of structures, it is necessary to integrate the equations of motion. A number of time integration schemes have been proposed for nonlinear dynamic analyses. Some of these schemes demand iterative calculation for each time step to compute the responses against dynamic loads, and others have the strict condition of analyses for the stability, such as the central difference method. A huge computing time is required for a nonlinear large degree-of-freedom system such as Finite Element analyses, especially, when the system shows such a strong non-linearity as the separation and sliding phenomena at the contact surface. Therefore, we propose a numerical integration scheme for nonlinear dynamic Finite Element Method.

In this paper we discuss the proposed numerical integration scheme. A brief description of the method and a few widely used time integration methods is presented and their application is illustrated. In addition, accuracy, stability, and efficiency of the proposed method are examined by the theoretical approach based on the modal analysis and the results for a example.

TIME INTEGRATION OF STRUCTURES WITH NONLINEAR STIFFNESS PROPERTIES

The equation of motion of a system with the nonlinear stiffness property can be written as

$$[m]\{\ddot{x}(t)\}+[c]\{\dot{x}(t)\}+[k]\{x(t)\}-\{q(t)\}=\{p(t)\} \qquad (1)$$

where $[m]$, $[c]$ and $[k]$ are mass, damping and stiffness matrices of the system, respectively, $\{x(t)\}$ nodal displacement vector, $\{q(t)\}$ a quasi-external force vector, which can be considered as an additional external forces to make the linear system equivalent to the nonlinear system, $\{p(t)\}$ external force vector and dot ($\cdot$) represents differential of time.

A time integration scheme requires that Eq.(1) should be satisfied at each discrete time. In this paper, the time step interval, Δt, is assumed to be constant and $t_k(=k\Delta t)$ is the time of kth-step, $\{x\}_k, \{\dot{x}\}_k, \{\ddot{x}\}_k$ nodal displacement, velocity and acceleration vectors, respectively, $\{q\}_k$ quasi-external forces and $\{p\}_k$ external forces, at time of t_k.

In this section, three time integration schemes, the iterative method, the central difference method and Sun's method are reviewed at first. Next we propose a new method for time integration of nonlinear multi-degree system which is suitable for Nonlinear Dynamic Finite Element analyses. In the explanation, $\{x\}$ denotes the responses of the entire system and $\{y\}$ the responses excited only by the increments of the quasi-external forces, $\{\Delta q\}$.

Iterative Method

This method is most popular for nonlinear Finite Element analyses, also called " Load Transfer method" (Toki et. al., 1980). This method is based on the averaged acceleration method for the time integration. The averaged acceleration method assumes the following relations;

$$\{\bar{\dot{x}}\}_{n+1} = \{\dot{x}\}_n + \frac{\Delta t}{2}(\{\ddot{x}\}_n + \{\bar{\ddot{x}}\}_{n+1}), \qquad \{\bar{x}\}_{n+1} = \{x\}_n + \Delta t\{\dot{x}\}_n + \frac{\Delta t^2}{4}(\{\ddot{x}\}_n + \{\bar{\ddot{x}}\}_{n+1}) \tag{2a,2b}$$

in which $^{-}$ means the responses at time t_{n+1} which are excited by $\{q\}_n$ instead of $\{q\}_{n+1}$ in Eq.(1), whose detail is described later. Eqs.(2a,2b) are solved for $\{\bar{\ddot{x}}\}_{n+1}$ and $\{\bar{\dot{x}}\}_{n+1}$ in terms of $\{\bar{x}\}_{n+1}$ as

$$\{\bar{\ddot{x}}\}_{n+1} = -\{\ddot{x}\}_n - \frac{4}{\Delta t}\{\dot{x}\}_n + \frac{4}{\Delta t^2}(\{\bar{x}\}_{n+1} - \{x\}_n), \qquad \{\bar{\dot{x}}\}_{n+1} = -\{\dot{x}\}_n + \frac{2}{\Delta t}(\{\bar{x}\}_{n+1} - \{x\}_n). \tag{3a,3b}$$

Substituting $\{\bar{\ddot{x}}\}_{n+1}$ and $\{\bar{\dot{x}}\}_{n+1}$ of Eqs.(3a,3b), respectively, into Eq.(1) at time t_{n+1}, we get $\{\bar{x}\}_{n+1}$ by solving the equation as follows;

$$\{\bar{x}\}_{n+1} = \left([k] + \frac{2}{\Delta t}[c] + \frac{4}{\Delta t^2}[m]\right)^{-1}\left(\{p\}_{n+1} + \{q\}_n + [m]\left(\frac{4}{\Delta t^2}\{x\}_n + \frac{4}{\Delta t}\{\dot{x}\}_n + \{\ddot{x}\}_n\right) + [c]\left(\frac{2}{\Delta t}\{x\}_n + \{\dot{x}\}_n\right)\right) \tag{4}$$

where the quasi-external force vector $\{q\}_n$ should be $\{q\}_{n+1}$ to satisfy Eq.(1) at time t_{k+1}, but we use $\{q\}_n$, because $\{q\}_{n+1}$ is unknown at the time of calculating $\{\bar{x}\}_{n+1}$. Thus $^{-}$ denotes that the responses are calculated with $\{q\}_n$ instead of $\{q\}_{n+1}$. We shall remove it when the responses satisfies Eq.(1). $\{q\}_{n+1}$ and $\{\bar{q}\}_{n+1}$ are quasi-external force vectors computed by $\{x\}_{n+1}$ and $\{\bar{x}\}_{n+1}$, respectively.

Substituting the quasi-external force vector $\{\bar{q}\}_{n+1}$ which is determined the nodal displacement vector $\{\bar{x}\}_{n+1}$ obtained from Eq.(4), into $\{q\}_n$ of Eq.(4), we get modified $\{\bar{x}\}_{n+1}$ and $\{\bar{q}\}_{n+1}$. The iterative method is required to repeat the calculations above until quasi-external forces converges. This method is so accurate that it is used in general, but the computing time for the iteration is very large. For example especially, in the case of analysis of separation by making use of joint element, it takes a great number of calculating time costs.

Central Difference Method

In the Central difference method, accelerations and velocities at time t_n are described, respectively, as follows;

$$\{\ddot{x}\}_n = \frac{1}{\Delta t^2}(\{x\}_{n+1} - 2\{x\}_n + \{x\}_{n-1}), \qquad \{\dot{x}\}_n = \frac{1}{2\Delta t}(\{x\}_{n+1} - \{x\}_{n-1}). \tag{5a,5b}$$

Substituting Eqs.(5a,5b) into Eq.(1) at time t_n, we get the equation below;

$$\{x\}_{n+1} = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\left(\{p\}_n\Delta t^2 + \{q\}_n\Delta t^2 - ([k]\Delta t^2 - 2[m])\{x\}_n - \left([m] - \frac{\Delta t}{2}[c]\right)\{x\}_{n-1}\right). \tag{6}$$

As Eq.(6) gives displacement time series, accelerations and velocities are obtained by substituting displacements into Eqs.(5a,5b), if necessary.

This method is very accurate in calculating with a sufficient small time interval, Δt, whereas the condition for it's analytical stability is highly strict. So the method is not suitable for analyses of the multi-degree-of-freedom such as Finite Element analysis, because Δt must be set very small value in order to satisfy the stable condition for it's highest mode.

Sun's Method

This method is based on the Wilson θ method for the time integration. The responses excited by the increments of quasi-external forces are independently inferred using the principle of impulse-momentum. (Sun et. al., 1991). This method is precise for a small degree-of-freedom system, while it is not suitable for a multi-degree-of-freedom system by the same reason with the central difference method. As mentioned above, this method is mainly based on the Wilson θ method (SUN(β=1/6, θ=1.38)), it can be extend to the linear acceleration method, (SUN(β=1/6, θ=1.0)) and the averaged acceleration method, (SUN(β=1/4, θ=1.0)). In the extended Sun's method for the averaged acceleration method, responses at time t_{n+1} are given by

$$\{x\}_{n+1} = \left([k] + \frac{2}{\Delta t}[c] + \frac{4}{\Delta t^2}[m]\right)^{-1}\left(\{p\}_{n+1} + \{q\}_n + [m]\left(\frac{4}{\Delta t^2}\{x\}_n + \frac{4}{\Delta t}\{\dot{x}\}_n + \{\ddot{x}\}_n\right) + [c]\left(\frac{2}{\Delta t}\{x\}_n + \{\dot{x}\}_n\right)\right) \quad (7)$$

$$\{\dot{x}\}_{n+1} = -\{\dot{x}\}_n + \frac{2}{\Delta t}(\{x\}_{n+1} - \{x\}_n) + [m]^{-1}\{\Delta q\}\frac{\Delta t}{2} \quad (8)$$

$$\{\ddot{x}\}_{n+1} = [m]^{-1}(\{p\}_{n+1} + \{q\}_{n+1} - [c]\{\dot{x}\}_{n+1} - [k]\{x\}_{n+1}). \quad (9)$$

The extended Sun's method is, as shown by numerical examples later, so stable in high frequency range that it can obtain a rather accurate solution for a multi-degree-of-freedom system, such as Finite Element analysis.

Proposed Method

In this paper, we propose a further stable time integration scheme for higher mode. In the proposed method, computing each response vector from Eqs.(3a,3b) and (4) in the same way as the iterative method, now we introduce the increment of the quasi-external force vector, $\{\Delta q\}(=\{q\}_{n+1} - \{q\}_n)$ and the responses excited by $\{\Delta q\}$ are calculated using the central difference method instead of iteration in the iterative method.

Now, suppose another system loaded only $\{\Delta q\}$. In case that $\{\Delta q\}$ is loaded at 1st-step, the equilibrium equation of this system can be written as

$$[m]\{\ddot{y}\}_1 + [c]\{\dot{y}\}_1 + [k]\{y\}_1 = \{\Delta q\} \quad (10)$$

where $\{y\}$ is nodal displacement vector excited by the increment of the quasi-external force vector. In the central difference method, the relations for $\{\ddot{y}\}_1$, $\{\dot{y}\}_1$ and $\{y\}_1$ are assumed by substituting n=1 into Eqs.(5a,5b). In which $\{y\}_1$ is $\{0\}$, because $\{y\}_1$ is calculated at the step before $\{\Delta q\}$ is loaded, and initial displacements, $\{y\}_0$ is also $\{0\}$. Then, we have

$$\{\ddot{y}\}_1 = \frac{1}{\Delta t^2}\{y\}_2, \qquad \{\dot{y}\}_1 = \frac{1}{2\Delta t}\{y\}_2. \quad (11a,11b)$$

Substituting the relations for $\{y\}_1$ and $\{y\}_2$ from Eqs.(11a,11b) into Eq.(10), we get Eq.(12).

$$\{y\}_2 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}\Delta t^2 \quad (12)$$

$\{\ddot{y}\}_1$ and $\{\dot{y}\}_1$ are derived by substituting Eq.(12) into Eqs.(11) as follows;

$$\{\ddot{y}\}_1 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}, \qquad \{\dot{y}\}_1 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}\frac{\Delta t}{2} \quad (13a,13b)$$

where $\{\ddot{y}\}_1$ and $\{\dot{y}\}_1$ are the acceleration and the velocity vectors, respectively, excited by $\{\Delta q\}$ at the step $\{\Delta q\}$ applied. On the other hand, Eqs.(3a,3b) and (4) can be written about the responses without considering $\{\Delta q\}$. Thus the summation of these two equations of motions for each systems satisfies Eq.(1) at time t_{n+1} for entire system. Then we get the responses at time t_{n+1} as follows;

$$\{x\}_{n+1} = \left([k] + \frac{2}{\Delta t}[c] + \frac{4}{\Delta t^2}[m]\right)^{-1}\left(\{p\}_{n+1} + \{q\}_n + [m]\left(\frac{4}{\Delta t^2}\{x\}_n + \frac{4}{\Delta t}\{\dot{x}\}_n + \{\ddot{x}\}_n\right) + [c]\left(\frac{2}{\Delta t}\{x\}_n + \{\dot{x}\}_n\right)\right) \quad (14)$$

$$\{\dot{x}\}_{n+1} = -\{\dot{x}\}_n + \frac{2}{\Delta t}(\{x\}_{n+1} - \{x\}_n) + \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}\frac{\Delta t}{2} \quad (15)$$

$$\{\ddot{x}\}_{n+1} = -\{\ddot{x}\}_n - \frac{4}{\Delta t}\{\dot{x}\}_n + \frac{4}{\Delta t^2}(\{x\}_{n+1} - \{x\}_n) + \left(\{m\} + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}. \tag{16}$$

As the response vectors in Eqs.(14), (15) and (16) satisfy Eq.(1) at time t_{n+1}, we don't put $^{-}$ on them. In above equations, $\{\Delta q\}$ is not $(\{\overline{q}\}_{n+1} - \{q\}_n)$ but $(\{q\}_{n+1} - \{q\}_n)$ because $\{y\}_1=\{0\}$ makes $\{x\}_{n+1}$ equal to $\{\overline{x}\}_{n+1}$ and $\{q\}_{n+1}$ equal to $\{\overline{q}\}_{n+1}$. Consequently, $\{q\}_{n+1}$ is determined by $\{x\}_{n+1}$ in Eq.(14).

PROPERTY OF PROPOSED METHOD

The proposed method is different from the other time integration method in dealing with the increment of quasi-external forces, which occurred in calculation of a nonlinear system. In this section, we consider the system loaded only the increments of quasi-external force vector, $\{\Delta q\}$. Then we clarify the property by comparing the response displacement vectors computed by the central difference method and the proposed method at the step next to the step at which $\{\Delta q\}$ is loaded. $\{x\}$ and $\{y\}$ are denoted by the response displacement vectors calculated by the proposed method and the central difference method, respectively, excited by the increments of quasi-external forces.

Consider a system of which initial, 0th-step, condition is that displacements, velocities and accelerations take zero value. $\{\Delta q\}$ is loaded at 1st-step. Then,

$$\{x\}_0 = \{\dot{x}\}_0 = \{\ddot{x}\}_0 = \{0\}, \quad \{p\}_0 = \{p\}_1 = \{p\}_2 = \{0\}, \quad \{q\}_0 = \{0\}, \quad \{q\}_1 = \{\Delta q\}. \tag{17}$$

In the proposed method, substituting $n = 0$ and Eqs.(17) into Eqs.(14), (15) and (16), we get the 1st-step's displacements, velocities and accelerations as follows;

$$\{x\}_1 = \{0\}, \quad \{\dot{x}\}_1 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}\frac{\Delta t}{2}, \quad \{\ddot{x}\}_1 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}. \tag{18}$$

Then, $n = 1$ and Eqs.(17) and (18) are substituted to obtain the response displacement vector at 2nd-step;

$$\{x\}_2 = \left([k] + \frac{2}{\Delta t}[c] + \frac{4}{\Delta t^2}[m]\right)^{-1}\left(\{\Delta q\} + [m]\left(\frac{4}{\Delta t^2}\{x\}_1 + \frac{4}{\Delta t}\{\dot{x}\}_1 + \{\ddot{x}\}_1\right) + [c]\left(\frac{2}{\Delta t}\{x\}_1 + \{\dot{x}\}_1\right)\right). \tag{19}$$

By substituting Eqs.(18) into Eq.(19) and factoring $([m] + [c]\Delta t/2)^{-1}\{\Delta q\}$, the different equation for $\{x\}_2$ is written as follows;

$$\{x\}_2 = \left([k] + \frac{2}{\Delta t}[c] + \frac{4}{\Delta t^2}[m]\right)^{-1}(4[m] + \Delta t[c])\left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}. \tag{20}$$

On the other hand, $\{y\}_2$ denotes the response displacement vector obtained by the central difference method at the 2nd-step under the same condition. Substituting $n = 1$ and Eqs.(17) into Eq.(6), we obtain $\{y\}_2$ as follows;

$$\{y\}_2 = \left([m] + \frac{\Delta t}{2}[c]\right)^{-1}\{\Delta q\}\Delta t^2. \tag{21}$$

Now we discuss the difference between $\{x\}_2$ and $\{y\}_2$. Substituting Eq.(21) to Eq.(20), we can get the relation between $\{x\}_2$ and $\{y\}_2$ as follows;

$$\{x\}_2 = (\Delta t^2[k] + 2\Delta t[c] + 4[m])^{-1}(4[m] + \Delta t[c])\{y\}_2. \tag{22}$$

In case of a Rayleigh hypothesis damping, eigen vector of the system must be orthogonal. The stiffness, damping and mass matrices can be written using eigen vector as follows;

$$\{\phi\}_i^T[k]\{\phi\}_j = \omega_i^2\delta_{ij}, \quad \{\phi\}_i^T[c]\{\phi\}_j = 2h_i\omega_i\delta_{ij}, \quad \{\phi\}_i^T[m]\{\phi\}_j = \delta_{ij}, \quad (1 \le i,j \le N) \tag{23}$$

where $\{\phi\}_i$ and h_i are the eigen vector and the damping factor for the eigen frequency, ω_i, respectively, and N the number of freedom and δij Kronecker's delta. We define $[\Phi]$ as the modal matrix of which ith-column component is eigen vector, $\{\phi\}_i$. In matrix form, Eqs.(23) is rewritten as

$$[\Phi]^T[m][\Phi] = [I] \tag{24}$$

The different formula of Eq.(22) is obtained using modal matrix $[\Phi]$ as follows;

$$\begin{aligned}\{x\}_2 &= [\Phi]\left([\Phi]^{-1}(\Delta t^2[k] + 2\Delta t[c] + 4[m])^{-1}([\Phi]^T)^{-1}\right)\left([\Phi]^T(4[m] + \Delta t[c])[\Phi]\right)[\Phi]^{-1}\{y\}_2 \\ &= [\Phi][A][\Phi]^{-1}\{y\}_2\end{aligned} \tag{25}$$

where,

$$[A]=[A_1]^{-1}[A_2],\quad [A_1]=[\Phi]^T(\Delta t^2[k]+2\Delta t[c]+4[m])[\Phi],\quad [A_2]=[\Phi]^T(4[m]+\Delta t[c])[\Phi]. \tag{26}$$

From Eqs.(23) and (26), A_{ij}, ith-row and jth-column component of $[A]$, is given by

$$A_{ij}=\frac{2h_i\omega_i\Delta t+4}{\omega_i^2\Delta t^2+4h_i\omega_i\Delta t+4}\delta_{ij},\quad (1\le i,j\le N). \tag{27}$$

By substituting $[\Phi]^{-1}$ obtained from Eq.(24) into Eq.(25), we get

$$\{X\}_2=[B][M]\{Y\}_2 \tag{28}$$

where,

$$[B]=[\Phi][A][\Phi]^T \qquad \text{or} \qquad B_{ij}=\sum_k^N\sum_l^N \Phi_{ik}A_{kl}\Phi_{lj}^T \tag{29}$$

where subscripts i and j denote a row and a column of matrix, respectively. Since [A] is a diagonal matrix as shown in Eq.(27), $A_{kl}=0$ in case of $k\neq l$, B_{ij} is given by

$$B_{ij}=\sum_k^N \Phi_{ik}A_{kk}\Phi_{kj}^T. \tag{30}$$

From Eq.(29), it seems that B_{ij} is under the influence of all component of [A]. But in fact, as Eq.(30) shows, A_{nn}, nth-row and nth-column diagonal component of $[A]$, has an effect upon B_{ij} only with Φ_{in} and Φ_{nj}^T. Where Φ_{in} and Φ_{jn} are ith- and jth-component of nth-eigen vector, respectively, therefore A_{nn} is not effective for B_{ij} without nth-eigen vector. And Eq.(21) shows that A_{ii} approximates to zero in higher modes and to 1 in lower modes. The relation between A_{ii} and eigen period are shown in Fig.1. In the figure, T denotes eigen period. This figure indicates that A_{ii} is about 0.9 at eigen period of $T/\Delta t=10$, approximates to 1 in lower modes and decrease radically to zero in higher modes than that period. Hence it is shown that the proposed method gives response displacements excited by the increments of quasi-external force, which are close to the displacements obtained by the central difference method in lower frequency and are smaller than those in higher frequency. This is the reason why the proposed method is accurate and stable.

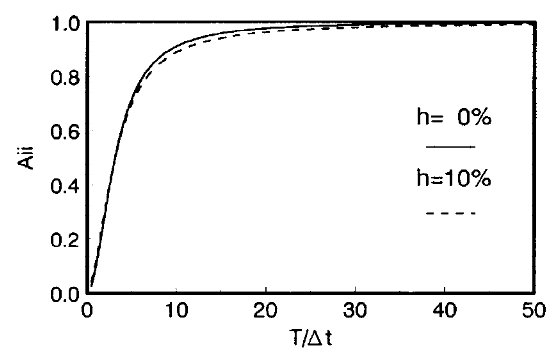


Fig.1 Relation between Aii and eigen period

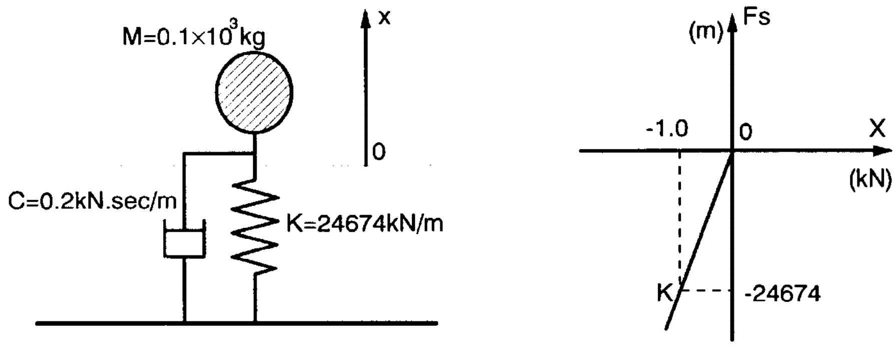
NUMERICAL EXAMPLE AND DISCUSSION

Two numerical examples are presented to verify the accuracy and the stability of the proposed method. Six time integration methods are compared by the single-degree-of-freedom free-fall and bound model (Model 1) and the dynamic soil-structure interaction model using FEM (Model 2). Two cases of time intervals are analyzed for comparison, since the accuracy and the stability of the solutions are influenced by the ratio of the computing time interval, Δt, to the eigen period of a system.

Six time integration methods used in the comparison are listed as follows;

- *Iterative Method*
- *Central Difference Method*
- Sun's Methods (*Sun*(β=1/6, θ=1.38), *Sun*(β=1/6, θ=1.0) and *Sun*(β=1/4, θ=1.0))
- *Proposed Method.*

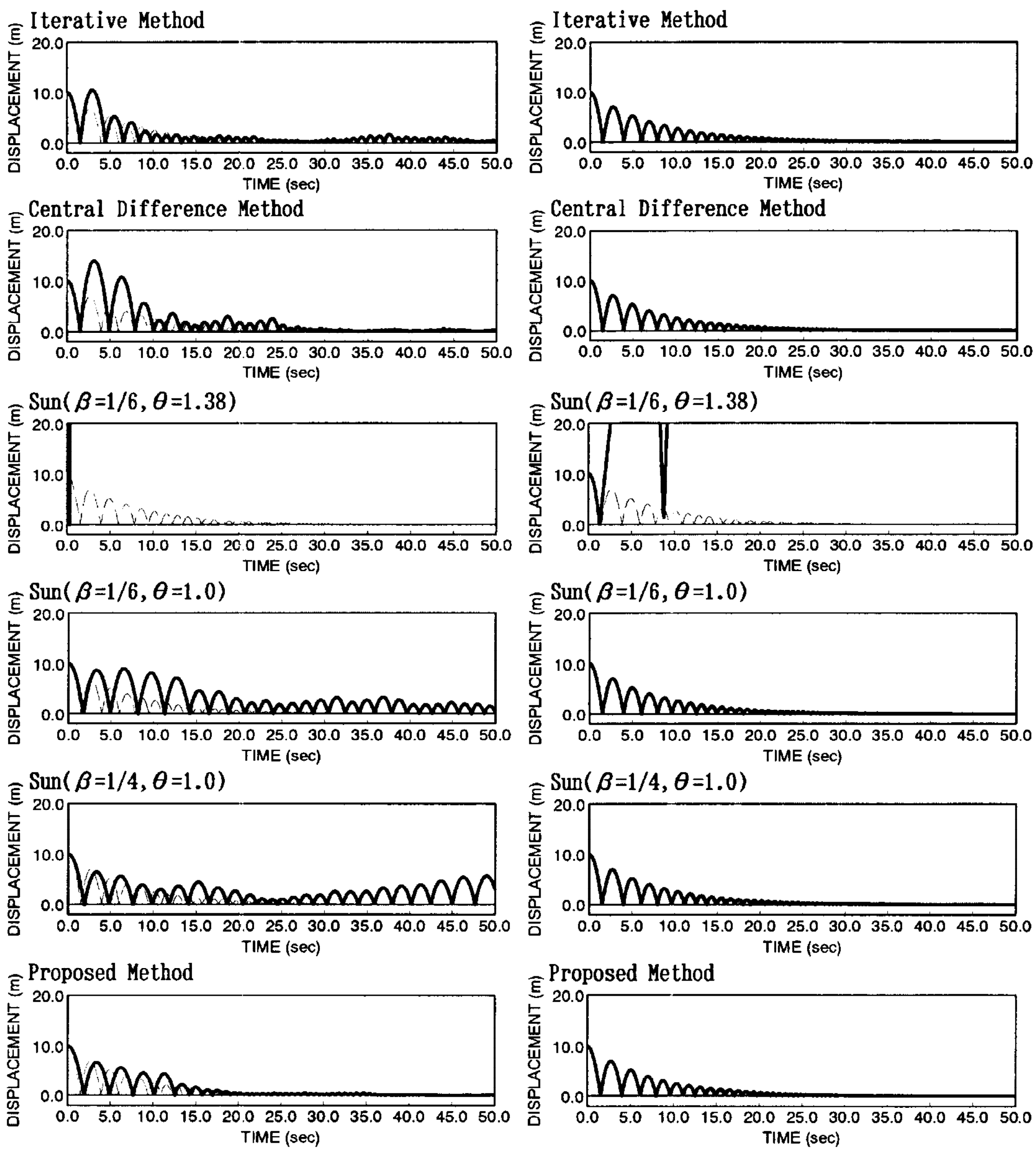
Single-degree-of-freedom Free-fall and Bound Model (Model 1)



(a) Single-degree-of-freedom free-fall and bound model

(b) Elasto-plastic stiffness

Fig.2 Single-degree-of-freedom of free-fall and bound model (Model 1)



(a) Δ t=T/4 (1/100 sec)

(b) Δ t=T/16 (1/400 sec)

Fig.3 Calculate displacement of mass (Model 1)

Suppose that a ball free-falls and bounds on a floor. In the model, the mass is supported by the damper and the spring as shown in Fig.2(a), and it may move only vertically. The gravity (Mg) is always acted vertically as an external force. The stiffness of the spring has a elasto-perfect plastic property as shown Fig.2(b). In case of $x \le 0$, the eigen period of this system is 0.04 second. The initial condition of this model is $x_0 = 10(m)$, $\dot{x}_0 = 0(m/\sec)$ and $\ddot{x}_0 = 0(m/\sec^2)$.

Fig.3 shows the time history of response displacement of the mass. In this figure, a dashed line indicates the response computed by *Iterative Method* with $\Delta t = T/32$. In case of $\Delta t = T/16$, the solution by Sun's method(β=1/6, θ=1.38) diverges, while those by the other methods give good approximation. On the other hand in the case of $\Delta t = T/4$, the solution by *Proposed Method* is stable but numerical errors make the natural period longer, while those by the others are unstable.

Finite Element Model of Soil-Structure System (*Model 2*)

Fig.4 shows the model 2, which is a structure (40m high and 12m width) resting on a half-space ground (320m width and 50m depth). The joint elements are adopted at the contact surface between soil and structure to represent separation and sliding. The shear constitutive relation of the joint element and soil element are assumed to be elasto-perfect plastic and the Mohr-Coulomb's law is adopted for the failure criteria. Table 1 shows physical properties of this model. In this model, the first eigen period is 1.3 second and the highest eigen period is 7×10^{-4} second. The input motion of the analyses is NS component of the El Centro (1940) of which peak acceleration is scaled to 200cm/sec^2.

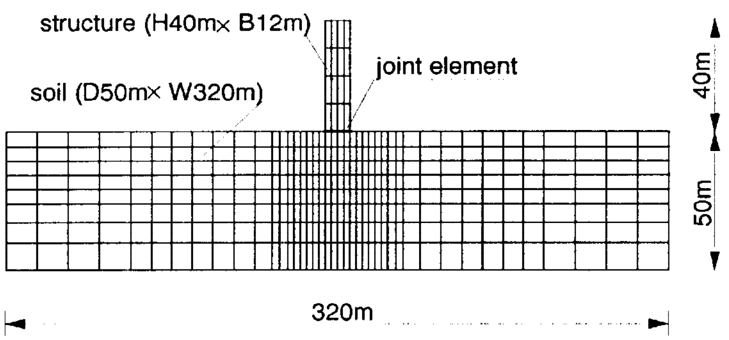


Fig.4 Finite element mesh of dynamic soil-structure interaction system (Model 2)

Table.1 Physical properties of system

	Soil	Structure	Joint
Unit weight (×9.8kN/m³)	1.8	2.3	ks=49 (GN/m)
Shear wave velocity (m/s)	286	2000	kn=49 (GN/m)
Poisson's ratio	0.30	0.16	
Damping factor	0.08	0.05	
Friction angle(°)	36		24
Cohesion (kN/m²)	0		0

Fig.5 shows the horizontal displacement of structure at the center of gravity in case of $\Delta t = 1/1{,}000$ and 1/200 second. In this figure, a dashed line denotes the response displacement computed by *Iterative Method* with Δt =1/10,000 second, while all methods give accurate solutions in the case of Δt =1/10,000 second. The figure indicates that Sun's method(β=1/4, θ=1.0) is accurate in the case of Δt =1/1,000 second but unstable in the case of Δt =1/200 second, while *Proposed Method* keeps high accuracy in the case of Δt =1/200 second. Sun's method for β=1/6, θ=1.38 and that for β=1/6, θ=1.0, and *Central Difference Method* are divergent in the case of Δt =1/2,000 and Δt =1/1,000 second. The necessary CPU time of *Iterative Method* for the analysis is about 15 and 70 times larger than that of *Proposed Method* in the case of Δt =1/1,000 and Δt =1/200 second, respectively. The CPU time of *Iterative Method* depends on its convergent condition, which is assumed in the analysis that the iterative increment of a nodal quasi-external force is less than 9.8×10^{-8} kN in this study.

CONCLUSIONS

A new time integration scheme for a large degree-of-freedom system with nonlinear stiffness such as Finite Element model has been developed. Theoretical approach based on the modal analysis and numerical analyses are conducted to examine the usefulness and validity of the proposed method in nonlinear dynamic response analyses. The conclusions are summarized as follows;

1. The extended Sun's method which is based on the average acceleration method is useful for nonlinear dynamic Finite Element Method.
2. The proposed method which is based on the average acceleration method and the central difference method is more stable than the extended Sun's method. The method is especially useful for response analysis of large degree-of-freedom such as Finite Element model.

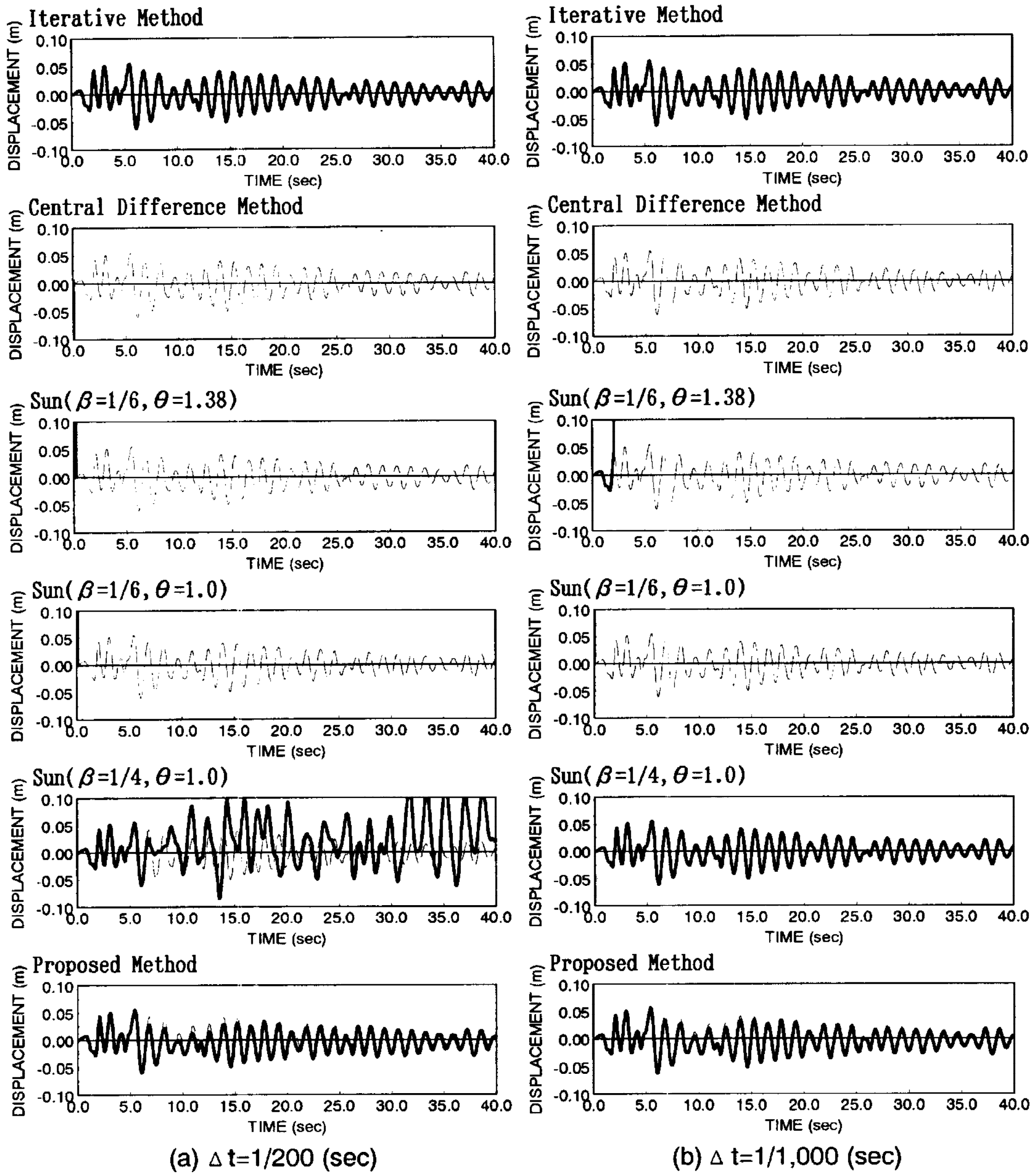


Fig.5 Horizontal displacement of structure at the center of gravity (Model 2)

REFERENSES

A.F.D' Souza and V.K. Garg : *Advanced dynamics, modeling and analysis*, Prentice-Hall, pp.213-227, 1984.

K. Sun and J.A. Pires : A Post-correction integration for nonlinear dynamic analysis of structures, *Earthquake eng. struct. dyn.* 20, pp.1083-1097, 1991.

T. Toki, T. Sato and F. Miura : Separation and sliding between soil and structure during strong ground motion, *Proc. JSCE*, No. 302, pp.31-41, 1980 (in Japanese).