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Non-iterative time integration scheme for non-linear dynamic FEM analysis

Riki Honda[1,*,†], Hisakazu Sakai[2] and Sumio Sawada[1]

[1]*Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan*
[2]*Earthquake Disaster Mitigation Research Center, National Research Institute for Earth Science and Disaster Prevention, 2465-1 Mikiyama, Miki, Hyogo 673-0433, Japan*

SUMMARY

This paper proposes a non-iterative time integration (NITI) scheme for non-linear dynamic FEM analysis. The NITI scheme is constructed by combining explicit and implicit schemes, taking advantage of their merits, and enables stable computation without an iteration process for convergence even when used for non-linear dynamic problems. Formulation of the NITI scheme is presented and its stability is studied. Although the NITI scheme is not unconditionally stable when applied to non-linear problems, it is stable in most cases unless stiffness hardening occurs or the problem has a large velocity-dependent term.

The NITI scheme is applied to dynamic analysis of the non-linear soil–structure system and computation results are compared with those by the central difference method (CDM). Comparison shows that the stability of the NITI scheme is superior to that of the CDM. Accuracy of the NITI scheme is verified because its results are identical with those by the CDM in which the time step is set as 1/10 of that for the NITI scheme. The application of the NITI scheme to the mesh-partitioned FEM is also proposed. It is applied to dynamic analysis of the linear soil–structure system. It yields the same results as a conventional single-domain FEM analysis using the Newmark β method. This result verifies the usability of mesh-partitioned FEM analysis using the NITI scheme. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: time integration scheme; non-iterative scheme; non-linear dynamic analysis; mesh-partitioned FEM

1. INTRODUCTION

With the rapid growth of computation power, large-scale non-linear dynamic analysis is gaining importance in the field of earthquake engineering and the demand for large and

*Correspondence to: Riki Honda, Disaster Prevention Research Institute, Kyoto University, Gokasho, Uji, Kyoto 611-0011, Japan.
†E-mail: honda@catfish.dpri.kyoto-u.ac.jp

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complex computation is also increasing. The computation scheme plays an important role in determining the efficiency of computation. If the time integration scheme of a non-linear dynamic analysis is stable and accurate, the time step can be set large and the computation time can be reduced. Otherwise, the analysis may just take a lot of computing time and end up with inaccurate results.

One of the popular time integration schemes used to numerically solve such problems is the central difference method (CDM), which is an explicit scheme and does not require iteration. However, the CDM is unstable compared to implicit schemes and in order to avoid instability the time step must be considerably small, which ends up with a long computation time. Implicit integration schemes, such as the Newmark β method [1] are stable and the time step can be relatively large. When the implicit time integration scheme is applied to non-linear problems, it requires iteration for convergence at every time step and again it takes a long computation time.

In order to take advantage of these schemes, some integration schemes, which combine the implicit and explicit time integration schemes, have been proposed such as a predictor–corrector method [2], an operator splitting method [3], or schemes that are made by modifying these schemes [4, 5].

In this paper we propose a new non-iterative time integration scheme (NITI scheme), which is applicable to a non-linear dynamic FEM analysis. The NITI scheme is based on an implicit scheme and accompanied by an explicit scheme. It is highly stable and can conduct efficient computation without an iteration process for convergence. A conventional implicit time integration scheme requires iteration for the convergence, but the NITI scheme replaces the iteration process with an explicit scheme. A similar time integration scheme is proposed by Sun *et al.* [6] and Sakai *et al.* [7, 8]. Their methods, however, are applicable only to the problem whose non-linearity is a function of displacement. They are not applicable to the problems whose non-linearity is dependent on velocity. The NITI scheme is developed by extending the scheme proposed by Sakai *et al.* and it is also applicable to the problems with velocity-dependent nonlinearity.

In the first part of this paper, the algorithm of the NITI scheme is described and formulation of the scheme is presented in detail. The Newmark β method is used as an example of an implicit time integration scheme, and the CDM is an example of an explicit scheme. They are combined in the computation process of the NITI scheme. Stability of the NITI scheme is also discussed based on the stability of a single-degree-of-freedom (SDOF) system, which corresponds to the modal analysis.

It is followed by a section in which we study the usability of the proposed NITI scheme by applying it to the dynamic analysis of a non-linear soil–structure system. The procedure to apply the NITI scheme to the non-linear problems is also described in detail. After that, we propose to use the NITI scheme in the dynamic analysis of the mesh-partitioned FEM, which was once used to combine the implicit and explicit time integration schemes [9–11] and is often used for the purpose of parallel computation. The mesh-partitioned FEM, which divides the model into subdomains, must deal with interaction between the subdomains. Interaction is an implicit function of state variables and therefore computation is often conducted by explicit schemes [12]. It will be presented that the NITI scheme can treat this interaction efficiently by regarding it as a non-linear external force. The efficiency of the NITI scheme applied to these computations will be discussed based on the computation results of these analyses.

2. NON-ITERATIVE TIME INTEGRATION SCHEME

2.1. Algorithm

In this section we describe the algorithm of the proposed non-iterative time integration (NITI) scheme, taking the ordinary equation of motion as an example. We use

$$M\ddot{x} + C^{NL}(t,x,\dot{x})\dot{x} + K^{NL}(t,x,\dot{x})x = p(t) \tag{1}$$

where t denotes time; x and p denote the displacement vector and external force vector, respectively; a dot over the variable denotes differentiation with respect to time, indicating that $\dot{x}$ and $\ddot{x}$ are velocity and acceleration, respectively; M, $C^{NL}(t,x,\dot{x})$ and $K^{NL}(t,x,\dot{x})$ denote mass, damping and stiffness matrices. If C^{NL} and K^{NL} are constant, Equation (1) represents a *linear* system. It will be called a *non-linear* system when stiffness, damping or both matrices are functions of displacement and velocity. If the damping matrix C is constant and the stiffness matrix is not dependent on velocity $\dot{x}$, time integration can be efficiently conducted by using the scheme presented by Sakai *et al.* [7, 8]. We extend the scheme by Sakai *et al.* and develop a scheme applicable to non-linear systems described by Equation (1).

We can rewrite Equation (1) without loss of generality as

$$M\ddot{x} + C\dot{x} + Kx = p(t) + f(t,x,\dot{x}) \tag{2}$$

by introducing a function f to consider the effect of non-linearity as

$$f(t,x,\dot{x}) = C\dot{x} - C^{NL}(t,x,\dot{x})\dot{x} + Kx - K^{NL}(t,x,\dot{x})x \tag{3}$$

where C and K are constant matrices.

Let us describe the computation process of the NITI scheme. Equation (2) is discretized in the time domain, assuming the time step is Δt. In the following, subscripts denote the time level. For example, x_n denotes the displacement x at time $t = t_n = n\Delta t$. We consider the process to update the time level from t_n to t_{n+1}. The equation of motion at time $t = t_{n+1}$ can be expanded as

$$\begin{aligned} M\ddot{x}_{n+1} + C\dot{x}_{n+1} + Kx_{n+1} &= p(t_{n+1}) + f(t_{n+1},x_{n+1},\dot{x}_{n+1}) \\ &= p(t_{n+1}) + f(t_{n+1},x_n,\dot{x}_n) + \Delta f_x + \Delta f_{\dot{x}} \end{aligned} \tag{4}$$

where

$$\Delta f_x = f(t_{n+1},x_{n+1},\dot{x}_n) - f(t_{n+1},x_n,\dot{x}_n) \tag{5}$$

$$\Delta f_{\dot{x}} = f(t_{n+1},x_{n+1},\dot{x}_{n+1}) - f(t_{n+1},x_{n+1},\dot{x}_n) \tag{6}$$

In the NITI scheme, response to the first two terms $p(t_{n+1}) + f(t_{n+1},x_n,\dot{x}_n)$ of Equation (4) is calculated by an implicit time integration scheme. Various schemes such as the Wilson θ method are available as an implicit scheme. In this paper, we use the Newmark β method ($\beta = 1/4$) as an implicit scheme. This process gives the solution of the equation of motion

$$M\ddot{x}_{n+1} + C\dot{x}_{n+1} + Kx_{n+1} = p(t_{n+1}) + f(t_{n+1}, x_n, \dot{x}_n) \tag{7}$$

In this process, since the function f to consider the non-linear behavior of the system is not yet updated, it is equivalent to solving the equation of motion as

$$M\ddot{x}_{n+1} + C^{NL}(t_{n+1}, x_n, \dot{x}_n)\dot{x}_{n+1} + K^{NL}(t_{n+1}, x_n, \dot{x}_n)x_{n+1} = p(t_{n+1}) \tag{8}$$

This solution includes inconsistency because it is calculated using the stiffness and damping matrices estimated from x_n and $\dot{x}_n$ of the previous time level $t = t_n$, instead of x_{n+1} and $\dot{x}_{n+1}$ at the target time level $t = t_{n+1}$. It is necessary to compensate for the gap by adding the difference of response due to the third and fourth terms of the right-hand side of Equation (4).

For this compensation, the ordinary implicit time integration scheme requires iteration until displacement and velocity values converge at every time level. If the non-linearity of the system is severe, this process can take considerable computation time.

In the NITI scheme, the response to the third term Δf_x and the fourth term $\Delta f_{\dot{x}}$ of the right-hand side of Equation (4) are estimated by using a central difference method (CDM). It is an explicit scheme and therefore, as will be shown later, does not require iteration for convergence. Summation of the responses to these terms $p + f, \Delta f_x$ and $\Delta f_{\dot{x}}$ gives the solution for the current time level $t = t_{n+1}$.

Let us describe the computation process in further detail. Firstly, we get the response to the first two terms of Equation (4), $p(t_{n+1}) + f(t_{n+1}, x_n, \dot{x}_n)$, using the Newmark β method ($\beta = 1/4$), as

$$\begin{aligned}\tilde{x}_{n+1} = {} & \left(K + \frac{2}{\Delta t}C + \frac{4}{\Delta t^2}M\right)^{-1} \\ & \times \left\{ f(t_{n+1}, x_n, \dot{x}_n) + M\left(\frac{4}{\Delta t^2}x_n + \frac{4}{\Delta t}\dot{x}_n + \ddot{x}_n\right) + C\left(+\frac{2}{\Delta t}x_n + \dot{x}_n\right)\right\}\end{aligned} \tag{9}$$

$$\dot{\tilde{x}}_{n+1} = -\dot{x}_n + \frac{2}{\Delta t}(\tilde{x}_{n+1} - x_n) \tag{10}$$

$$\ddot{\tilde{x}}_{n+1} = -\ddot{x}_n - \frac{4}{\Delta t}\dot{x}_n + \frac{4}{\Delta t^2}(\tilde{x}_{n+1} - x_n) \tag{11}$$

where the symbol ˜ denotes a tentatively updated value. It would be worth mentioning that all the coefficient matrices that appear in Equations (4) to (6) are constant matrices and do not require updating at all, although we are dealing with a non-linear problem. They are calculated once at the beginning of the computation and the same matrices can then be used for the rest of the computation.

Response to the third term Δf_x and the fourth term $\Delta f_{\dot{x}}$ of Equation (4) can be estimated as the response of the system to the impulsive forces Δf_x and $\Delta f_{\dot{x}}$ applied at time $t = t_{n+1}$, assuming that the system is at a static state before the impulsive force is applied. It means that displacements of the system are assumed to be zero for $t \leqslant t_{n+1}$. In the NITI scheme, for the purpose of avoiding iteration for convergence, these responses are calculated by an explicit scheme, CDM.

Let us consider the case where the impulsive force p_{n+1} is applied at time $t = t_{n+1}$. In the CDM, acceleration and velocity are approximated as

$$\ddot{x}_{n+1} = \frac{1}{\Delta t^2}(x_{n+2} - 2x_{n+1} + x_n) \tag{12}$$

$$\dot{x}_{n+1} = \frac{1}{2\Delta t}(x_{n+2} - x_n) \tag{13}$$

Substituting them in

$$M\ddot{x}_{n+1} + C\dot{x}_{n+1} + Kx_{n+1} = p_{n+1} \tag{14}$$

gives:

$$M\left(\frac{x_{n+2} - 2x_{n+1} + x_n}{\Delta t^2}\right) + C\left(\frac{x_{n+2} - x_n}{2\Delta t}\right) + Kx_{n+1} = p_{n+1} \tag{15}$$

Since the displacement of the system is assumed to be 0 for $t \leqslant t_{n+1}$, we have an initial condition

$$x_{n+1} = x_n = 0 \tag{16}$$

Solving Equation (15) with respect to x_{n+2} yields

$$x_{n+2} = \left(M + C\frac{\Delta t}{2}\right)^{-1} p_{n+1}\Delta t^2 \tag{17}$$

From Equations (12) and (13), we obtain

$$\dot{x}_{n+1} = \left(M + C\frac{\Delta t}{2}\right)^{-1} p_{n+1}\frac{\Delta t}{2} \tag{18}$$

$$\ddot{x}_{n+1} = \left(M + C\frac{\Delta t}{2}\right)^{-1} p_{n+1} \tag{19}$$

A set of solutions given above in Equations (16), (18) and (19) is the response to the impulsive force. Replacing $p_{n+1} = \Delta f_x$, we obtain compensating terms as

$$\Delta^x x_{n+1} = 0 \tag{20}$$

$$\Delta^x \dot{x}_{n+1} = \left(M + C\frac{\Delta t}{2}\right)^{-1} \Delta f_x \frac{\Delta t}{2} \tag{21}$$

$$\Delta^x \ddot{x}_{n+1} = \left(M + C\frac{\Delta t}{2}\right)^{-1} \Delta f_x \tag{22}$$

where Δ^x indicates that they are the terms to compensate for the difference due to the update of displacement x.

If a function f in the right-hand side of Equation (4) is independent of velocity, Equation (6) gives $\Delta f_{\dot{x}} = 0$. In such a case, we get the solution for the time level $t = t_{n+1}$ just by adding each of Equations (20)–(22) to each of Equations (9)–(11), respectively. Since displacement of the response in Equation (20) is zero, there is no need to update the displacement x_{n+1} and right-hand side of Equation (4) does not require further updating.

If f is dependent on velocity, we move on to the next step and estimate the response to the increment of the external force $\Delta f_{\dot{x}_{n+1}}$. It should be noticed that $\Delta f_{\dot{x}_{n+1}}$ of Equation (6) must be estimated using $\dot{x}_{n+1}$ which is obtained by taking the summation of Equations (10) and (21). The response acceleration is obtained by using the central difference approximation using velocity $\dot{x}$, instead of displacement x, which gives

$$\ddot{x}_{n+1} = \frac{1}{2\Delta t}(\dot{x}_{n+2} - \dot{x}_n) \tag{23}$$

We can also assume

$$x_{n+1} = 0 \tag{24}$$

$$\dot{x}_{n+1} = \dot{x}_n = 0 \tag{25}$$

Substituting Equations (23)–(25) into the equation of motion (14) gives

$$x_{n+2} = 2\Delta t M^{-1} p_{n+1} \tag{26}$$

Replacing $p_{n+1} = \Delta f_{\dot{x}}$, the response to $\Delta f_{\dot{x}}$ can be obtained as

$$\Delta^{\dot{x}} x_{n+1} = 0 \tag{27}$$

$$\Delta^{\dot{x}} \dot{x}_{n+1} = 0 \tag{28}$$

$$\Delta^{\dot{x}} \ddot{x}_{n+1} = M^{-1} \Delta f_{\dot{x}} \tag{29}$$

where $\Delta^{\dot{x}}$ indicates the terms to compensate for the difference due to the update of velocity $\dot{x}$. Here let us again emphasize that the coefficient matrices such as $(M + C\frac{\Delta t}{2})^{-1}$ and M^{-1} are constant and they do not require updating during the computation process.

Adding each result of Equations (27)–(29) to the corresponding results of Equations (20)–(22) and Equations (9)–(11), we get the solution for the time level $t = t_{n+1}$. Since the velocity response (27) and displacement response (28) against the increment of $\Delta f_{\dot{x}}$ are zero, there is no need to update the velocity and displacement and no further computation is required. The solution for time level $t = t_{n+1}$ is obtained as

$$x_{n+1} = \tilde{x}_{n+1} \tag{30}$$

$$\dot{x}_{n+1} = \tilde{\dot{x}}_{n+1} + \Delta^{x} \dot{x}_{n+1} \tag{31}$$

$$\ddot{x}_{n+1} = \tilde{\ddot{x}}_{n+1} + \Delta^{x} \ddot{x}_{n+1} + \Delta^{\dot{x}} \ddot{x}_{n+1} \tag{32}$$

It should be noted that the acceleration, velocity and displacement obtained as Equations (30)–(32) satisfy the equation of motion (1) at every time level.

2.2. Stability of the NITI scheme

This section discusses the stability of the NITI scheme. When the NITI scheme is applied to a linear problem, it is identical with the Newmark β method and therefore it is unconditionally stable when $\beta \geqslant 1/4$. Let us consider the case in which the proposed method is applied to a non-linear problem. We consider the case where damping or stiffness of the system (or both) are dependent on the displacement and velocity.

We can investigate the stability of the NITI scheme by tracing the behavior of free oscillation of a single-degree-of-freedom (SDOF) system computed by the NITI scheme. This is based on the idea of modal analysis. The SDOF system corresponds to one of the modes of the total system. Let us assume the SDOF system as

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{33}$$

where m, c and k are mass, damping and stiffness, respectively. We introduce the parameters θ_c and θ_k, to rewrite Equation (33) as

$$m\ddot{x} + (1 - \theta_c)c\dot{x} + (1 - \theta_k)kx = -\theta_c c\dot{x} - \theta_k kx \tag{34}$$

We consider the left-hand side of Equation (34) as an equation of motion and the right-hand side as an external force. Since the external force is given as a function of displacement and velocity, the NITI scheme is applicable. It should be remembered that Equations (33) and (34) represent identical systems no matter what values θ_c and θ_k may take.

The parameter θ_c determines how the system is dependent on the velocity, and the parameter θ_k shows how it depends on displacement. Dependence on velocity and displacement can be adjusted by changing the parameters θ_c and θ_k.

When we consider the left-hand side of Equation (34) as an equation of motion of the system, which we tentatively call the *initial* system, the stiffness of this system is $(1 - \theta_k)k$. However, the actual restoring force applied to the system is equivalent to $(1 - \theta_k)kx + \theta_k kx = kx$, because Equation (34) is identical with Equation (33). This means that the restoring force is larger than that of the initial system when $\theta_k > 0$. Therefore, we can interpret that $\theta_k > 0$ corresponds to the case in which stiffness hardening occurs. Obviously $\theta_k < 0$ means softening of the stiffness.

Discretization of the equation of motion in the time domain using the Newmark β method yields

$$m\ddot{x}_{n+1} + (1 - \theta_c)c\dot{x}_{n+1} + (1 - \theta_k)kx_{n+1} = f_{n+1} \tag{35}$$

$$\dot{x}_{n+1} = \dot{x}_n + \tfrac{1}{2}\ddot{x}_n\Delta t + \tfrac{1}{2}\ddot{x}_{n+1}\Delta t \tag{36}$$

$$x_{n+1} = x_n + \dot{x}_n\Delta t + \left(\tfrac{1}{2} - \beta\right)\ddot{x}_n\Delta t^2 + \beta\ddot{x}_{n+1}\Delta t^2 \tag{37}$$

where Δt denotes the time step and f denotes the right-hand side of Equation (34) as

$$f_{n+1} = -\theta_c c\dot{x}_{n+1} - \theta_k kx_{n+1} \tag{38}$$

First, we solve Equation (34) for the time level $t = t_{n+1}$. The right-hand side of the equation is a function of displacement and velocity, but their values at time $t = t_{n+1}$ are still unknown at this stage. Therefore, in the right-hand side of Equation (34), we use the value of velocity and displacement at the previous time level $t = t_n$, instead of the current time level $t = t_{n+1}$. We replace the right-hand side of Equation (35) by

$$f_n = -\theta_c c\dot{x}_n - \theta_k k x_n \tag{39}$$

and rewrite Equations (35)–(37) in the matrix form as:

$$A\tilde{X}_{n+1} = BX_n \tag{40}$$

where the symbol $\tilde{}$ means that the value is tentative and requires updating later, and

$$X_n = \begin{Bmatrix} \ddot{x}_n \\ \dot{x}_n \\ x_n \end{Bmatrix}, \quad \tilde{X}_{n+1} = \begin{Bmatrix} \tilde{\ddot{x}}_{n+1} \\ \tilde{\dot{x}}_{n+1} \\ \tilde{x}_{n+1} \end{Bmatrix} \tag{41}$$

$$A = \begin{bmatrix} m & (1-\theta_c)c & (1-\theta_k)k \\ -\frac{1}{2}\Delta t & 1 & 0 \\ -\beta\Delta t^2 & 0 & 1 \end{bmatrix} \tag{42}$$

$$B = \begin{bmatrix} 0 & -\theta_c c & -\theta_k k \\ \frac{1}{2}\Delta t & 1 & 0 \\ (\frac{1}{2}-\beta)\Delta t^2 & \Delta t & 1 \end{bmatrix} \tag{43}$$

Therefore, the tentatively updated state vector is given as

$$\tilde{X}_{n+1} = A^{-1}BX_n \tag{44}$$

Next, we consider the response to Δf_x, which is a compensation of the external force caused by update of displacement from time level $t = t_n$ to $t = t_{n+1}$. Correction $\Delta^x X$ due to the difference of displacement is given by substituting

$$\Delta f_x = -\theta_k k \Delta x \tag{45}$$

$$\Delta x = \tilde{x}_{n+1} - x_n \tag{46}$$

in Equations (20)–(22). We get

$$\Delta^x X = \begin{Bmatrix} \Delta^x \ddot{x} \\ \Delta^x \dot{x} \\ \Delta^x x \end{Bmatrix} = -\left\{ m + (1-\theta_c)c\frac{\Delta t}{2} \right\}^{-1} \theta_k k \begin{Bmatrix} 1 \\ \Delta t/2 \\ 0 \end{Bmatrix} \Delta x \tag{47}$$

which can be written as

$$\Delta^x X_{n+1} = H_x(\tilde{X}_{n+1} - X_n) \tag{48}$$

where

$$H_x = -\left\{ m + (1-\theta_c)c\,\frac{\Delta t}{2} \right\}^{-1} \theta_k k \cdot \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & \Delta t/2 \\ 0 & 0 & 0 \end{bmatrix} \tag{49}$$

Correcting terms to compensate for the difference due to the update of velocity, $\Delta^{\dot{x}} X_{n+1}$, is obtained in a similar manner. Substituting

$$\Delta f_{\dot{x}} = -\theta_c c \Delta \dot{x} \tag{50}$$

$$\Delta \dot{x} = \tilde{\dot{x}}_{n+1} + \Delta^x \dot{x}_{n+1} - \dot{x}_n \tag{51}$$

in Equations (27)–(29) gives

$$\Delta^{\dot{x}} X = \begin{Bmatrix} \Delta^{\dot{x}} \ddot{x} \\ \Delta^{\dot{x}} \dot{x} \\ \Delta^{\dot{x}} x \end{Bmatrix} = -m^{-1} \theta_c c \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \Delta \dot{x} \tag{52}$$

It should be noticed that $\Delta \dot{x}$ used in these equations includes $\tilde{\dot{x}}_{n+1}$ obtained from Equation (44) and $\Delta^x \dot{x}_{n+1}$ obtained from Equation (48). It can be expressed as

$$\Delta^{\dot{x}} X_{n+1} = H_{\dot{x}} (\tilde{X}_{n+1} - X_n + \Delta^x X_{n+1}) \tag{53}$$

where

$$H_{\dot{x}} = -m^{-1} \theta_c c \cdot \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{54}$$

Taking the summation of Equations (44), (48) and (53), we get a solution for time $t = t_{n+1}$ as

$$\begin{aligned} X_{n+1} &= \tilde{X}_{n+1} + \Delta^x X_{n+1} + \Delta^{\dot{x}} X_{n+1} \\ &= [A^{-1}B + H_x(A^{-1}B - I) + H_{\dot{x}}\{(A^{-1}B - I) + H_x(A^{-1}B - I)\}] X_n \end{aligned} \tag{55}$$

Let D denote the part in [ ] in Equation (55), then D can be regarded as an updating matrix of the system under consideration. Free oscillation of the system is obtained as

$$X_n = D^n X_0 \tag{56}$$

where X_0 is the initial state. Let λ_i denote the i-th eigenvalue of D and e_i a corresponding eigenvector. Expanding the initial state using eigenvectors as $X_0 = \sum e_i$, the state at time $t = n\Delta t$ grows to

$$X_n = D^n e_i = \sum \lambda_i^n e_i \tag{57}$$

This indicates that investigation of eigenvalues of D can show various aspects of the proposed time integration scheme including its stability. As for the stability, the largest absolute value of the eigenvalue is essential. If any of the eigenvalues of D has an absolute value larger than unity, the corresponding eigenvector component included in X_n diverges exponentially to infinity and the scheme becomes unstable. If all the eigenvalues have absolute values smaller than unity, the scheme is stable. We refer to the largest of the absolute values of eigenvalues as a *radius*. Stability of the proposed NITI scheme will be discussed based on the radius.

First we discuss the property of the NITI scheme, assuming $\theta_c = 0$. It corresponds to the case in which the system has displacement-dependent non-linearity but does not have velocity-dependent non-linearity. Parameters to represent normalized frequency, ω, and damping factor, h, are introduced as

$$\omega = \sqrt{\frac{k}{m}}\,\Delta t \tag{58}$$

$$h = \frac{c}{2\sqrt{mk}} \tag{59}$$

The radius is computed by changing θ_k from -2 to 0.5 and ω from 0 to 10π. $\theta_c = 0$ and $h = 0.01$ are assumed. $\theta_k = -2$ means that the stiffness decreases to 1/3 of the initial value and $\theta_k = 0.5$ means that the stiffness becomes twice as large as the initial value. $\omega = 0$ denotes the 0 Hz component and $\omega = 10\pi$ corresponds to ten times the Nyquist frequency ($10 \times 50\,\text{Hz} = 500\,\text{Hz}$ when the time step $\Delta t = 1/100\,\text{sec}$). Distribution of the radius is plotted in Figure 1. In the figure, a radius larger than unity is plotted as unity.

Figure 1 shows that the radius is smaller than unity for most of the area, indicating that the scheme is stable for most cases. The radius becomes larger than unity only in the area where ω is large and θ_k takes a positive value. A large ω corresponds to high frequency, and a positive θ_k to hardening of the stiffness. Figure 1 indicates that the computation process becomes unstable when stiffness hardening occurs. On the other hand, the radius consistently takes a value smaller than unity for the area where stiffness softens. It indicates that the

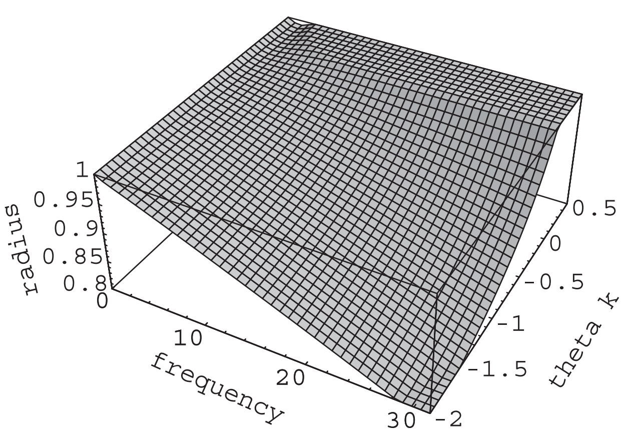


Figure 1. The radius for various frequencies ω and θ_k ($\theta_c = 0$ and $h = 0.01$).

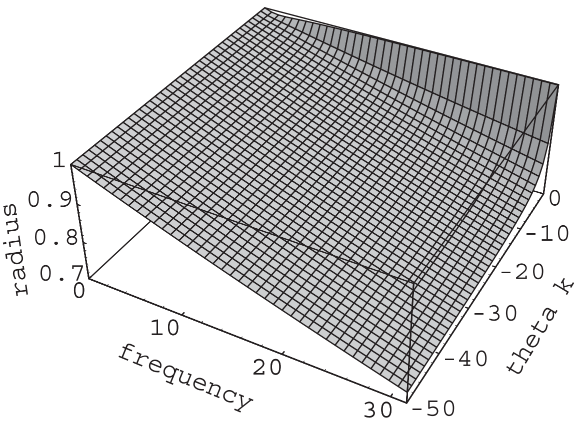


Figure 2. Distribution of the radius of the proposed scheme applied to the problem in which stiffness decreases to 1/50 of the initial value ($\theta_c = 0$ and $h = 0.01$).

proposed scheme can conduct stable computation for stiffness softening problems. In most of the dynamic analyses of the structures considered in earthquake engineering, structures suffer damage and their stiffness decreases. The NITI scheme can conduct stable computations for such analyses.

In order to investigate the stability of the NITI scheme applied to stiffness softening problems, the radius is calculated for θ_k in the range of -50 to 0, where $\theta_k = -50$ means the stiffness decreases to 2% of the initial value. Other conditions are the same as those for the above-mentioned case. Distribution of the computed radius is plotted in Figure 2. The results show that the radius is always smaller than unity, indicating that the proposed scheme is practically always stable when applied to the problem of stiffness softening.

Next we discuss the stability of the NITI scheme applied to the problem with velocity-dependent non-linearity. The radius is calculated assuming $\theta_c = 0.8$, which means that the damping increases to five times larger than the initial value. θ_k is changed from -2 to 0.5 and ω from 0 to 10π; $h = 0.01$ is assumed. Distribution of the radius is plotted in Figure 3. Distribution is quite similar to that in the case of $\theta_c = 0$ (Figure 1). The radius is less than unity for most of the case. It becomes larger than unity only when θ_k is larger than zero (stiffness hardening occurs). These results indicate that the NITI scheme is practically always stable unless the stiffness hardens from the initial value.

We also investigate the stability of the NITI scheme for various levels of the velocity-dependence of the non-linearity by changing the value of θ_c. The radius is estimated by changing θ_c from -2 to 0.5, which means that the damping changes from 1/3 to twice the initial value. The range of frequency ω is set from 0 to 5π. θ_k is fixed to -0.1, which means about 9% decrease of stiffness from the initial value. Distribution of the radius is plotted in Figure 4. The radius is smaller than unity in the whole plotted range. It means that the NITI scheme is stable even when velocity-dependence of the non-linearity becomes large.

On the other hand, however, it should also be recognized that computation by the NITI scheme can become unstable when the velocity-dependent non-linear term is large, even if the term is given as a damping term. Let us show an example in which damping factor is set

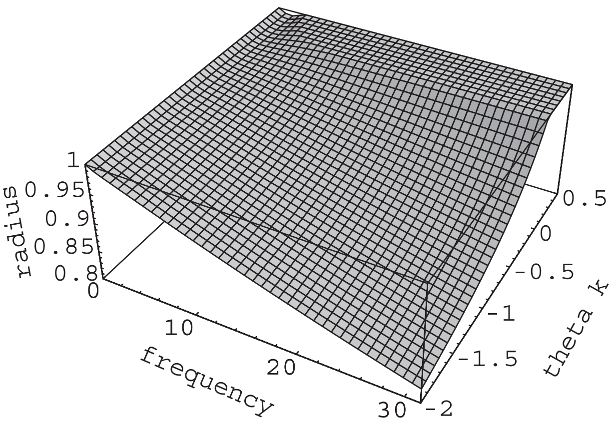


Figure 3. Distribution of the radius of the proposed scheme applied to the stiffness softening problem with velocity-dependent non-linearity. $\theta_c = 0.8$ and $h = 0.01$ are assumed.

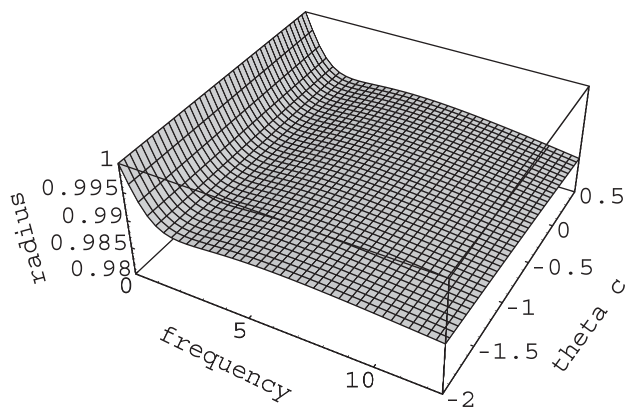


Figure 4. Distribution of the radius of the proposed scheme for various levels of velocity-dependent non-linearity. $\theta_k = -0.1$ (stiffness decrease), and $h = 0.01$ are assumed.

as large as $h = 1.0$ and the velocity-dependence parameter $\theta_c = 0.2$. Distribution of the radius for $\omega \in [0, 2\pi]$ and $\theta_k \in [-2, 0.5]$ is plotted in Figure 5. The radius becomes larger than unity in the high-frequency range. This indicates that computation of the NITI scheme may become unstable when the system has a large velocity-dependent term, such as a large damping term. Figure 5 also shows that θ_k scarcely affects the value of the radius. This means that such instability can happen no matter whether the stiffness may harden or soften. It should be noted that when the NITI scheme is applied to the practical problems, large damping may raise instability of computation instead of dampening and stabilizing the computation process.

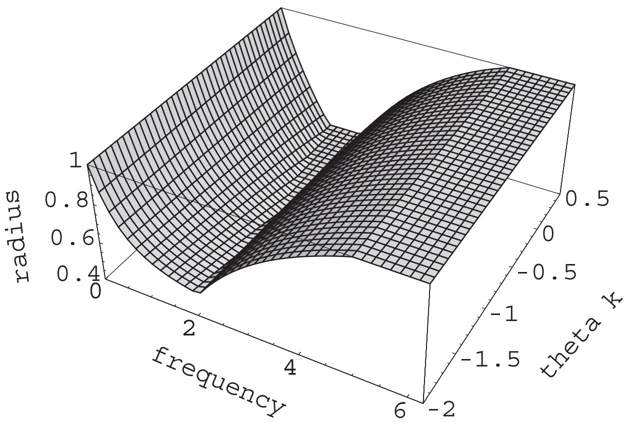


Figure 5. Relation of the radius, frequency ω and θ_k for the large damping factor $h = 1.0$; $\theta_c = 0.2$ is assumed.

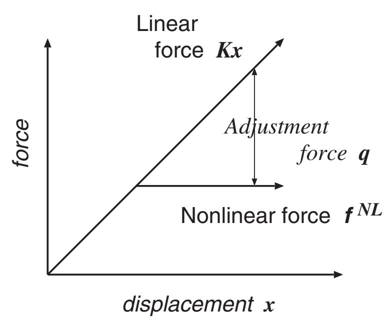


Figure 6. Adjustment force to consider the non-linear restoring force.

3. APPLICATION TO NON-LINEAR DYNAMIC FEM ANALYSIS

In order to study the usability of the NITI scheme, it is applied to the dynamic analysis of a non-linear FEM. Section 3.1 describes the computation process to treat the non-linear behavior of the soil. It is followed by Section 3.2 which presents computation results obtained by the NITI scheme. Accuracy of the NITI scheme is shown by comparing the results with those by the central difference method.

3.1. Computation process to consider non-linearity

The non-linear behavior of the elasto-plastic soil material that follows the Mohr–Coulomb's fracture criteria is implemented by using an adjustment force. The adjustment force q, which is illustrated in Figure 6, is defined as

$$q_n = Kx_n - f_n^{NL} \tag{60}$$

where the subscript n denotes the time level; x denotes the displacement; f^{NL} is a non-linear restoring force; and K is a constant stiffness matrix. The adjustment force q is applied as an external force in equation of motion (2). It can be seen that q corresponds to the last two terms of Equation (3).

Let us describe a scheme to consider the non-linearity. To avoid complexity, a simple constitutive rule for the non-linear behavior of the soil is adopted in the following FEM analysis. The force–displacement relationship in Equation (60) is calculated from the stress and strain which follow the Mohr–Coulomb's fracture criteria.

We consider the 2D problem and the stress vector Σ is introduced as

$$\Sigma = \{\sigma_1, \sigma_3, \tau_{13}\}^{\mathrm{T}} \tag{61}$$

where σ_1 and σ_3 are normal stresses and τ_{13} denotes tangential stress. First, the tentative increment of the stress vector $\Delta\tilde{\Sigma}$ in the element is evaluated from the increment of displacement of the nodes ΔX as

$$\Delta\tilde{\Sigma} = DB\Delta X \tag{62}$$

where D and B denote the material matrix and B matrix (strain–displacement matrix), respectively. Adding $\Delta\tilde{\Sigma}$ to the stress vector Σ_n at the current time level $t = t_n$, we obtain the tentative stress vector $\tilde{\Sigma}_{n+1}$ at the next time level $t = t_{n+1}$ as

$$\tilde{\Sigma}_{n+1} = \Sigma_n + \Delta\tilde{\Sigma} \tag{63}$$

If $\tilde{\Sigma}_{n+1}$ does not exceed the fracture criteria, it is adopted as the stress Σ_{n+1} at time $t = t_{n+1}$ as

$$\Sigma_{n+1} = \tilde{\Sigma}_{n+1} \tag{64}$$

and there is no need to update the adjustment force from the value given for the current time level, which means

$$q_{n+1} = q_n \tag{65}$$

If the stress condition exceeds the fracture criteria, the stress condition is shifted to the new stress condition $\bar{\Sigma}_{n+1}$ which satisfies the fracture criteria (see Figure 7). It is assumed, for simplicity, that when the stress condition is shifted, the principal stress direction and average principal stress are kept unchanged. The stress vector and adjustment force to be applied as a nodal force should be updated as

$$\Sigma_{n+1} = \bar{\Sigma}_{n+1} \tag{66}$$

$$q_{n+1} = q_n + \Delta q \tag{67}$$

$$\Delta q = \int B^{\mathrm{T}}(\tilde{\Sigma}_{n+1} - \bar{\Sigma}_{n+1})\,\mathrm{d}V = B^{\mathrm{T}}(\tilde{\Sigma}_{n+1} - \bar{\Sigma}_{n+1})V \tag{68}$$

where V denotes the volume of the element.

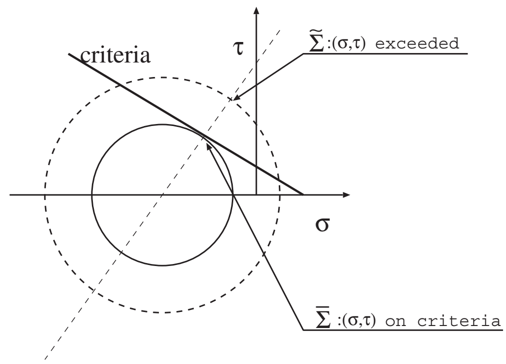


Figure 7. Stress used in the analysis of elasto-plastic material following Mohr–Coulomb's fracture criteria.

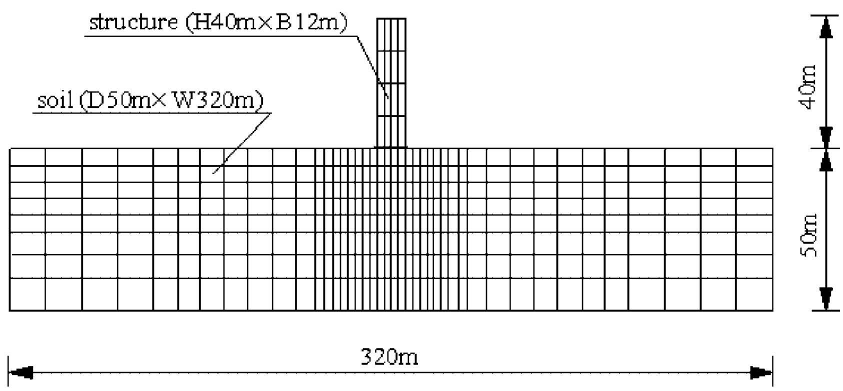


Figure 8. 2D soil–structure model.

3.2. Soil–structure system

The non-linear dynamic analysis of the 2D soil–structure system is conducted. The model consists of a structure and a ground. The structure is 40 m in height and 12 m in width, and the ground is 320 m wide and 50 m deep. The model is made using four-node isoparametric elements. It has 412 nodes and consists of 356 elements. The model is illustrated in Figure 8 and the properties assumed in the analysis are listed in Table I.

The structure is assumed to be a linear material. The soil is assumed to be an elasto-plastic material which yields according to the Mohr–Coulomb's fracture criteria. The Rayleigh damping $C=\alpha M+\beta K$ $(\alpha=0.361, \beta=0.00165)$ is added so that it gives the damping ratio $h=0.05$ at frequency 10.0 Hz.

In the numerical analysis, a seismic motion is applied as an inertial force. The NS component of the ground motion record obtained at Port Island [13] at the depth of 72 m during the 1995 Kobe Earthquake is used as an input motion. The time history of the input motion is shown in Figure 9.

Table I. Properties of the system.

Property	Soil	Structure
Unit weight [ton/m^3]	1.8	2.3
Shear wave velocity [m/sec]	286	2000
Poisson ratio	0.30	0.16
Damping factor[a]	0.05	0.05
Cohesion	0	—
Friction angle [degree]	36	—

[a]Damping at frequency of 10 Hz.

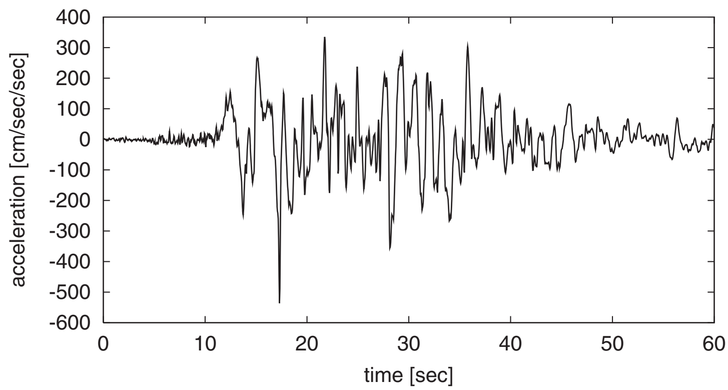


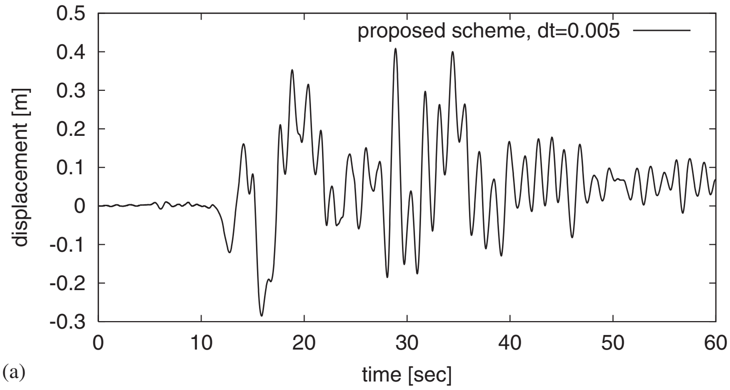
Figure 9. Input motion for dynamic analysis.

For the purpose of comparison, analysis is conducted by the NITI scheme and the central difference method (CDM). As shown in Table I, soil is assumed to have no cohesion and its fracture criteria is very small in the shallow part where there is little confining pressure. During the excitation by the earthquake ground motion, therefore, the shallow part of the ground is exposed to severe non-linearity and the computation becomes unstable. In order to avoid computational instability, we need to make the time step sufficiently small.

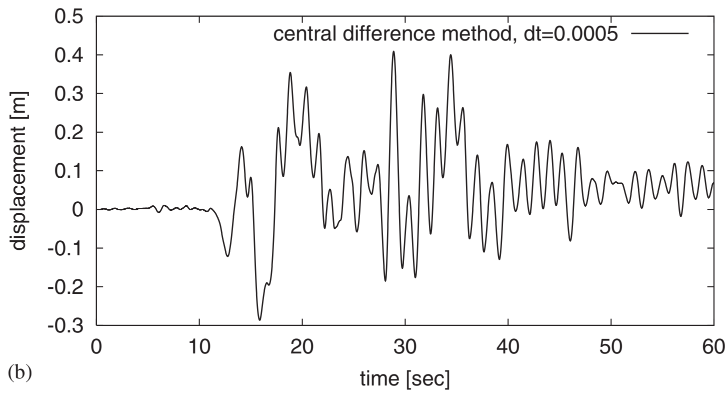
As for the CDM, the time step has to be taken as $\Delta t = 0.0005$ sec, since the computation diverges and the answer is not obtained when the time step is set as $\Delta t = 0.005$, 0.001 or 0.00075 sec. On the other hand, the NITI scheme can be conducted with the time step $\Delta t = 0.005$ sec. This indicates superiority of the NITI scheme over the CDM in terms of stability of computation.

The time histories of horizontal displacement at the gravity center of the structure, which are calculated by the CDM ($\Delta t = 0.0005$ sec) and by the NITI scheme ($\Delta t = 0.005$ sec), are plotted in (a) and (b) of Figure 10, respectively. They show that the NITI scheme and the CDM give identical results, verifying the accuracy of the NITI scheme.

Let us compare the computation time required by these analyses. All computations are conducted on a computer with a CPU of Pentium III 600 MHz. For the computation shown above, the NITI scheme requires a CPU time of 84.03 seconds, while the CDM requires 649.81 seconds. Although it can be expected that the computation time would be comparable if



(a)



(b)

Figure 10. Comparison of the computation results by the NITI scheme and the conventional central difference method: (a) NITI scheme ($\Delta t = 0.005$ sec); and (b) central difference method ($\Delta t = 0.0005$ sec).

we assume the same time step for both schemes, it should be recognized that the computation time is considerably reduced without losing computational accuracy because of the stability of the NITI scheme.

4. APPLICATION TO MESH-PARTITIONED FEM

This section introduces another application of the NITI scheme. We propose the application of the NITI scheme to mesh-partitioned FEM dynamic analysis, and verify its usability through numerical computation results. Firstly, the computation process of the mesh-partitioned FEM using the NITI scheme is described in Section 4.1. It is followed by Section 4.2 which compares the computation results obtained by mesh-partitioned FEM analysis using the NITI scheme and that by ordinary FEM analysis.

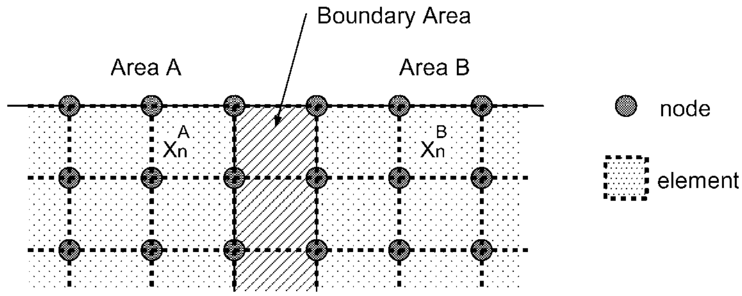


Figure 11. Partitioning of finite element meshes.

4.1. Computation process of mesh-partitioned FEM

This section describes the computation process to apply the NITI scheme to a mesh-partitioned FEM, which is one of the methods [12] used to treat the huge model by parallel computation. In mesh-partitioned FEM analysis, where the model is divided into small subdomains, interaction between subdomains is determined from the displacement and velocity of the subdomains. This indicates that by regarding the interaction between subdomains as a function of the state variables of subdomains, the NITI scheme can be applied to its dynamic analysis.

The computation process is described by considering the situation illustrated in Figure 11. The total model is divided into two areas; A and B. They are away from each other by one column of elements. All nodes, including those of the elements in the boundary area, belong either to the area A or to the area B. The superscript A or B denotes the area the node belongs to, and subscript n denotes the time level. For example, x_n^{A}, $\dot{x}_n^{\mathrm{A}}$ and $\ddot{x}_n^{\mathrm{A}}$ denote displacement, velocity and acceleration at time $t = t_n$ on the point belonging to the area A. In the same manner, x_n^{B}, $\dot{x}_n^{\mathrm{B}}$ and $\ddot{x}_n^{\mathrm{B}}$ denote the state variables of the node in the area B.

The equation of motion of the area A and B with an external force p, is given as

$$[M]\{\ddot{x}_n\} + [C]\{\dot{x}_n\} + [K]\{x_n\} = \{p_n\} \tag{69}$$

where

$$\{x_n\} = \begin{Bmatrix} x_n^{\mathrm{A}} \\ x_n^{\mathrm{B}} \end{Bmatrix}, \quad \{p_n\} = \begin{Bmatrix} p_n^{\mathrm{A}} \\ p_n^{\mathrm{B}} \end{Bmatrix} \tag{70}$$

$$[M] = \begin{bmatrix} M^{\mathrm{AA}} & M^{\mathrm{AB}} \\ M^{\mathrm{BA}} & M^{\mathrm{BB}} \end{bmatrix}, \quad [C] = \begin{bmatrix} C^{\mathrm{AA}} & C^{\mathrm{AB}} \\ C^{\mathrm{BA}} & C^{\mathrm{BB}} \end{bmatrix}, \quad [K] = \begin{bmatrix} K^{\mathrm{AA}} & K^{\mathrm{AB}} \\ K^{\mathrm{BA}} & K^{\mathrm{BB}} \end{bmatrix} \tag{71}$$

where the superscripts denote the area each matrix is related to. K^{AB}, for example, is the stiffness matrix connecting the displacement of the area B and the force applied to the area A. For simplicity, we consider the problem with lumped mass matrix and we have

$$M^{\mathrm{AB}} = M^{\mathrm{BA}} = O \tag{72}$$

The equation of motion of the area A can be written as

$$M^{AA}\ddot{x}_n^A + C^{AA}\dot{x}_n^A + K^{AA}x_n^A = p_n^A - C^{AB}\dot{x}_n^B - K^{AB}x_n^B \tag{73}$$

and that for the area B is also written in a similar manner as

$$M^{BB}\ddot{x}_n^B + C^{BB}\dot{x}_n^B + K^{BB}x_n^B = p_n^B - C^{BA}\dot{x}_n^A - K^{BA}x_n^A \tag{74}$$

The right-hand side of Equations (73) and (74) can be considered as the external force which is a function of the displacement and velocity of the areas A and B. They can be treated in the same manner as an external force is treated in the NITI scheme. It should be also noticed that the left-hand side of Equation (73) includes only state variables of the area A and Equation (74) includes those of the area B. It means that these equations can be solved independently.

Let us describe the detail of the computation process to update from time level $t = t_n$ to $t = t_{n+1}$:

1. Calculate the displacement, velocity and acceleration in the area A at time $t = t_{n+1}$, estimating the right-hand side of Equation (73), which is written as f^A, using the displacement and velocity of the area B at the current time level ($t = t_n$) as

$$f^A = p_{n+1}^A - C^{AB}\dot{x}_n^B - K^{AB}x_n^B \tag{75}$$

   The state variables at time level $t = t_{n+1}$ in the area B are also calculated in the same manner using the displacement and velocity of the area A at the current time level.
2. Updated displacement in the area B, x_n^B, gives the compensating factor Δf_x^A of the external force applied to the subdomain A as

$$\Delta f_x^A = -K^{AB}(x_{n+1}^B - x_n^B) \tag{76}$$

   We can estimate the response to this compensating force Δf_x^A using Equations (21) and (22). Remember that displacement does not require a further update. If there is no velocity-dependent damping in the boundary area ($C^{AB} = 0$), the updating process is completed here. State variables in the area B are also updated in the same manner.
3. If there is a damping term ($C^{AB} \neq 0$), we must take its effect into consideration. The increment of the velocity-dependent external force can be obtained as

$$\Delta f_{\dot{x}}^A = -C^{AB}(\dot{x}_{n+1}^B - \dot{x}_n^B) \tag{77}$$

   The response acceleration $\Delta^{\dot{x}}\ddot{x}$ to this increment of the external force can be obtained by substituting Equation (77) in Equation (29). Displacement and velocity do not require updating. By adding this response acceleration, the final value of the acceleration for the time level $t = t_{n+1}$ can be obtained. The same procedure should be taken for the area B.

So far, the computation process is described implicitly assuming that mesh-partitioning should be made following the alignment of element, as illustrated in Figure 11. However, as can be understood from the computation process, the way of mesh-partitioning can be independent of the physical configuration of the system. It is possible, for example, to assign horizontal and vertical components to different subdomains. This is advantageous because it allows us to decide mesh-partitioning purely by pursuing the efficiency of computation, without being constrained by the physical mesh configuration.

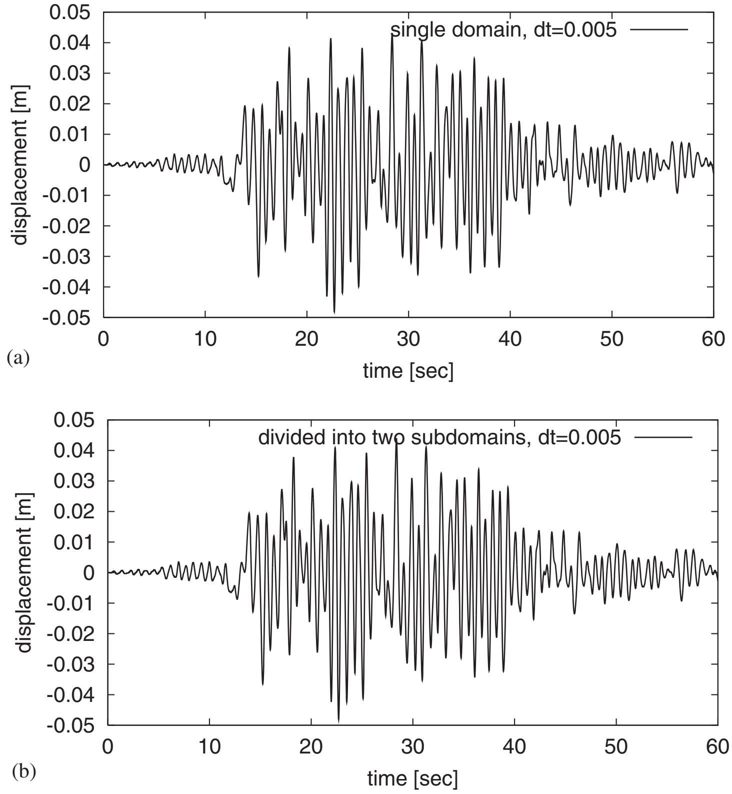


Figure 12. Comparison of the computation results by the FEM regarding the whole system as a single domain and that by the proposed scheme dividing the system into two subdomains ($\Delta t = 0.005$ sec): (a) FEM with a single domain; and (b) FEM with two subdomains.

4.2. Soil–structure system

The proposed scheme for the mesh-partitioned FEM is applied to the dynamic analysis of the 2D linear soil–structure model. The model is the same as illustrated in Figure 8 and Table I. Since the purpose of this computation is to verify the applicability of the NITI scheme to the mesh-partitioned FEM, we assume linear behavior for both the soil and the structure. The analysis is conducted using the same input motion as for the case in Section 3.2 (the strong motion record obtained at Port Island during the 1995 Kobe Earthquake). The model has a damping proportional to the mass matrix and the damping matrix is set as $C = \alpha M$ ($\alpha = 0.0361$).

Two cases of computation are conducted for comparison. In one case, the model is divided into two subdomains for mesh-partitioned FEM analysis, and in the other case, the whole system is treated as a single domain and treated by the conventional FEM. Computation of the mesh-partitioned FEM is conducted using the NITI scheme and that of the ordinary single domain FEM analysis is carried out using the Newmark β method. The time step is taken as $\Delta t = 0.005$ sec in both cases.

Time histories of displacement of the gravity center of the structure are calculated for the two cases and are plotted in Figure 12. They give identical results and this verifies the applicability of the NITI scheme to the dynamic analysis of the mesh-partitioned FEM. Some other computation results which verify the efficiency of the proposed mesh-partition FEM analysis are also previously reported [14].

5. CONCLUSION

This paper proposes a non-iterative time integration (NITI) scheme for the non-linear dynamic FEM analysis. The NITI scheme is developed by combining an implicit time integration scheme and an explicit scheme, taking advantage of their merits. The NITI scheme has two major advantages that enable efficient computation: one is that the NITI scheme does not require iteration for convergence even if it is applied to a non-linear problem; the other is that the stability of the NITI scheme is comparable with that of implicit schemes.

Formulation of the NITI scheme is presented using the Newmark β method as an implicit scheme and the central difference method (CDM) as an explicit method. The computation procedure of the NITI scheme is simple and straightforward. One can easily implement it in program codes by using the formulation presented in this paper.

The stability of the NITI scheme is also studied through investigation of the behavior of the SDOF system computed by the NITI scheme, which is based on the idea of modal analysis. The study reveals that computation of the NITI scheme is stable as long as stiffness hardening does not occur and the system does not have large velocity-dependent terms, such as a large damping term.

Two types of numerical examples are presented to show the usability of the NITI scheme. First, the NITI scheme is applied to a dynamic FEM analysis of the non-linear soil–structure system. The dynamic analysis is properly conducted by the CDM when the time step is set as $\Delta t = 0.0005$ sec, but the computation diverges when the time step is set larger than that. Dynamic analysis of the same problem is stably performed by the NITI scheme even if the time step is set as $\Delta t = 0.005$ sec. The computation results are identical with the results by the CDM with a time step $\Delta t = 0.0005$ sec. These results verify the stability and accuracy of the NITI scheme.

Secondly, application of the NITI scheme to the dynamic analysis of the mesh-partitioned FEM is proposed and the computation procedure is described. In the analysis of the mesh-partitioned FEM, which divides the whole model into subdomains to compute them separately, consideration of the interaction between subdomains is essential. It is presented that the NITI scheme can perform efficient computation by treating the interaction in the same manner as an external force of the non-linear problems. The computation results of the mesh-partitioned FEM using the NITI scheme are compared with the results by a conventional single-domain FEM analysis. These results show good agreement and the applicability of the NITI scheme to mesh-partitioned FEM analysis is verified.

It can be concluded that the efficiency and usability of the proposed NITI scheme are verified by these computation results. Our future work is to apply these computation schemes to various practical problems.

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